MATHCOUNTS®

You Don't Have to Solve for *x*!

Warm-Up!

Try these problems before watching the lesson.

Coach instructions: *Give students around 10 minutes (2 minutes per problem) to go through the warm-up problems.*

1. What is the value of $12(p + 9) - 12(p - 3)$?

   Distributing and solving, we get $12(p + 9) - 12(p - 3) = 12p + 12(9) - 12p + 12(3) = 12(9) + 12(3) = 12(9 + 3) = 12(12) =$ 144.

2. If $\frac{1}{6}$ of 400 equals $\frac{5}{6}$ of x, what is the value of x?

   Translating this into an algebraic equation, we get $(1/6)(400) = (5/6)x$. Multiplying both sides by 6/5 and solving for x we get $x = (6/5)(1/6)(400) = (1/5)(400) =$ 80.

3. If $5x - 4 = 26$, what is the value of x?

   Solving for x, we get $5x - 4 = 26 \rightarrow 5x = 30 \rightarrow x =$ 6.

4. If $6x + px = 14x$ and $x \neq 0$, what is the value of p?

   Factoring the left side of the equation, we can rewrite the equation as $(6 + p)x = 14x$. Dividing both sides by x, we get $6 + p = 14$ and $p = 14 - 6 =$ 8.

5. If $\frac{1}{b} = \frac{b}{a}$ and $b = -1$, what is the value of a?

   Substituting in -1 for b, we get $1/-1 = -1/a$. Solving for a, we get $-1 = -1/a \rightarrow a =$ 1.

The Problems

Coach instructions: *After students try the warm-up problems, play the video and have them follow along with the solutions.*

Take a look at the following problems and follow along as they are explained in the video.

6. If $3x + 1 = 10$, what is the value of $6x + 1$?

   Solution in video. Answer: 19.

7. Given that $x + \frac{1}{x} = 4$, what is the value of $x^4 + \frac{1}{x^4}$?

   Solution in video. Answer: 194.

Piece It Together

Coach instructions: *After watching the video, have student go back and finish any warm-up problems they didn't have time to complete or solved incorrectly. Then give students 10 to 15 minutes to try the next five problems.*

Note: *The terms in blue italics commonly appear in competition problems. Make sure Mathletes understand their meaning!*

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

8. If $3x + 155 = 272$, then what is the value of $3x + 160$?

Before solving for x, let's look at what we are being asked for. Notice that the value of $3x + 160$ is the same as $3x + 155 + 5$ so the answer will be $272 + 5 = 277$.

9. Given $7x + 13 = 328$, what is the value of $14x + 13$?

If we double both sides of the first equation we get $2(7x + 13) = 2(328) \rightarrow 14x + 26 = 656$. Subtracting 13 from both sides gives us $14x + 13 = 656 - 13 = 643$.

10. If $5x + 2 = 4.003$, what is the value of $20x + 7$? Express your answer as a decimal to the nearest ***thousandth***.

If we multiply both sides of the first equation by 4, we get $4(5x + 2) = 4(4.003) \rightarrow 20x + 8 = 16.012$. Subtracting 1 from each side gives us $20x + 7 = 15.012$.

11. If $x + \frac{1}{x} = 6$, what is the value of $x^3 + \frac{1}{x^3}$?

You may notice similarities between this problem and the second video problem. If we square both sides of the first equation, we get $x^2 + 2 + 1/x^2 = 36 \rightarrow x^2 + 1/x^2 = 34$. Next, if we take this new equation and multiply it by the first equation again, we get $(x + 1/x)(x^2 + 1/x^2) = (6)(34) \rightarrow x^3 + 1/x + x + 1/x^3 = 204$. Notice the middle two terms are the same as our original equation. Substituting in 6 for these terms and solving, we get $x^3 + 6 + 1/x^3 = 204 \rightarrow x^3 + 1/x^3 = 204 - 6 = 198$.

12. If a, b and c are ***integers*** such that $a + b = 6$, $b + c = 8$ and $a + c = 10$, what is the value of $a + b + c$?

Here we have three different variables, but we don't need to solve for their individual values. Let's add all three of our equations together. We get $(a + b) + (b + c) + (a + c) = 6 + 8 + 10$ or $2a + 2b + 2c = 24$. If we divide both sides by 2, we get our final answer $a + b + c = 12$.

Where are these questions from?

1. *From 2016 Chapter CDR, #70*
2. *From 2017 State CDR, #25*
3. *From 2014 Chapter CDR, #3*
4. *From 2014 Chapter CDR, #39*
5. *From 2016 Chapter Sprint, #3*
6. *From 2016 Chapter CDR, #21*
7. *From 2017 Chapter CDR, #40*
8. *From 2016 School Handbook, #8*
9. *From 2016 State Sprint, #3*
10. *From 2016 State CDR, #64*
11. *From 2014 State CDR, #14*
12. *From 2016 Chapter Sprint, #21**
**problem modified*

Optional Extension

Coach instructions: *Once your students have completed the problems and feel they have a comfortable understanding of the concept, let them try this fun extension activity. They might enjoy working on this in small groups.*

To extend your understanding and have a little fun with math, try the following activities.

Practice being a mathemagician; make people think you are a mind reader when you are just using your algebra skills! Here is an example of a "trick":

Think of a number between 1 and 50.
Double your number.
Add 6 to your new number.
Divide by 2.
Subtract your original number.
Is you new number....3?

The saying goes, "a magician never reveals the secret," but all you need is algebra to figure this out. Try translating each of the lines of the instructions of the trick into algebraic expressions with the original number as a variable to prove why it works. Try this trick on your friends or family members! Can you come up with more tricks of your own?

This is a great way to have Mathletes practice translating word problem into algebraic equations and to develop an understanding of the concept of inverse operations as a means to solve algebraic equations. Here is an explanation of the "trick":

The Magic Trick	The Algebra	The Explanation
Think of a number between 1 and 50.	N	Since we do not know what number the Mathlete picked, we use a variable to represent the number.
Double your number.	$2N$	Here we ask the Mathlete to double the number. We will later use inverse operations to undo this so we can cancel out the unknown.
Add 6 to your new number.	$2N + 6$	Adding 6 here will allow us to have a remainder (our "guess") when we cancel out the original number.
Divide by 2.	$(2N + 6) \div 2 = N + 3$	Dividing by 2 undoes the doubling from before.
Subtract your original number.	$N + 3 - N = 3$	Subtracting the original number cancels out the variable. The remaining number is the mathemagician's "guess."

Note: *This is a portion of an activity from a previous National Math Club book. If you would like the activity in it's entirety. Email cara@mathcounts.org.*